

EXERCISES (PART 1)

Exercise 1.1. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$.

1.1.1. Show that f is linear if and only if $f(1) = 1$.

Exercise 1.2. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$. Suppose that f is not linear. Show that f is not continuous at any point. (Hint: Use the fact that f is not linear to show that f is not continuous at any point.)

Exercise 1.3. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$. Suppose that f is not linear. Show that f is not continuous at any point.

Exercise 1.4. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$. Suppose that f is not linear. Show that f is not continuous at any point.

Exercise 1.5. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$.

Exercise 1.6.



Exercise 1.7.