

1. (10 points)

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that

$$f(x+y) = f(x) + f(y) \quad \forall x, y \in \mathbb{R}.$$

Prove that

- f is linear, i.e., $f(x) = cx$ for some constant $c \in \mathbb{R}$.
- f is continuous at 0 if and only if f is continuous at every point.
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2. (10 points)

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f\left(\frac{k}{n}\right) = \int_0^1 f(x) dx$$